EXAM IV
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. What is $\lim_{x\to 0}\left(\dfrac{\dfrac{1}{x-1}+1}{x}\right)$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) the limit does not exist

Ans

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2. $\displaystyle\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

(A) $\ln\sqrt{x}+C$ (B) $x+C$ (C) e^x+C (D) $\frac{1}{2}e^{2\sqrt{x}}+C$ (E) $e^{\sqrt{x}}+C$

Ans

3. If $y = \dfrac{3}{4 + x^2}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{3}{2x}$

(B) $\dfrac{3x}{\left(1 + x^2\right)^2}$

(C) $\dfrac{6x}{\left(4 + x^2\right)^2}$

(D) $\dfrac{-6x}{\left(4 + x^2\right)^2}$

(E) $\dfrac{-3}{\left(4 + x^2\right)^2}$

Ans

4. If $F(x) = \int_1^x (\cos 6t + 1)\, dt$, then $F'(x) =$

(A) $\sin 6x + x$ (B) $\cos 6x + 1$ (C) $\frac{1}{6}\sin 6x + x$

(D) $-\frac{1}{6}\sin 6x + 1$ (E) $\sin 6x + 1$

Ans

5. Consider the curve $x + xy + 2y^2 = 6$. The slope of the line tangent to the curve at the point $(2,1)$ is

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $-\frac{1}{3}$

(D) $-\frac{1}{5}$

(E) $-\frac{3}{4}$

Ans

6. $\lim_{h \to 0} \dfrac{3\left(\frac{1}{2} + h\right)^5 - 3\left(\frac{1}{2}\right)^5}{h} =$

(A) 0

(B) 1

(C) $\dfrac{15}{16}$

(D) the limit does not exist

(E) the limit can not be determined

Ans

7. If $p(x) = (x - 1)(x + k)$ and if the line tangent to the graph of p at the point $(4, p(4))$ is parallel to the line $5x - y + 6 = 0$, then $k =$

(A) 2

(B) 1

(C) 0

(D) –1

(E) –2

Ans

8. If $\cos x = e^y$ and $0 < x < \frac{\pi}{2}$, what is $\dfrac{dy}{dx}$ in terms of x ?

(A) $-\tan x$

(B) $-\cot x$

(C) $\cot x$

(D) $\tan x$

(E) $\csc x$

Ans

9. At $t = 0$, a particle starts at the origin with a velocity of 6 feet per second and moves along the x-axis in such a way that at time t its acceleration is $12t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 16 ft

(B) 20 ft

(C) 24 ft

(D) 28 ft

(E) 32 ft

Ans

10. When the area of an expanding square, in square units, is increasing three times as fast as its side is increasing, in linear units, the side is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 2 (E) 1

Ans

11. The average (mean) value of $\frac{1}{x}$ over the interval $1 \le x \le e$ is

(A) 1 (B) $\frac{1}{e}$ (C) $\frac{1}{e^2} - 1$ (D) $\frac{1+e}{2}$ (E) $\frac{1}{e-1}$

Ans

12. What is $\lim\limits_{x\to\infty} \dfrac{3x^2+1}{(3-x)(3+x)}$?

(A) -9 (B) -3 (C) 1 (D) 3 (E) The limit does not exist.

Ans $\square$

13. If $\displaystyle\int_{-2}^{2} (x^7 + k)\,dx = 16$, then $k =$

(A) -12

(B) 12

(C) -4

(D) 4

(E) 0

Ans $\square$

14. Consider the function f defined on $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ by $f(x) = \dfrac{\tan x}{\sin x}$ for all $x \ne \pi$. If f is continuous at $x = \pi$, then $f(\pi) =$

(A) 2

(B) 1

(C) 0

(D) -1

(E) -2

Ans $\square$

15. The function $f(x) = x^4 - 18x^2$ has a relative minimum at $x =$

(A) 0 and 3 only

(B) 0 and -3 only

(C) -3 and 3 only

(D) 0 only

(E) $-3,\ 0,\ 3$

Ans

16. The graph of $y = 3x^5 - 10x^4$ has an inflection point at

(A) $(0, 0)$ and $(2, -64)$

(B) $(0, 0)$ and $(3, -81)$

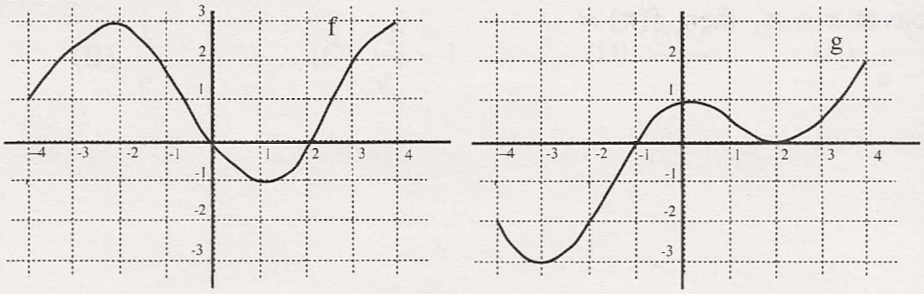
(C) $(0, 0)$ only

(D) $(-3, 81)$ only

(E) $(2, -64)$ only

Ans

17. The composite function h is defined by $h(x) = f[g(x)]$, where f and g are functions whose graphs are shown below.



The number of horizontal tangent lines to the graph of h is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Ans

18. The region in the first quadrant bounded by the graph of $y = \text{Arcsin}\, x$, $y = \frac{\pi}{2}$ and the y-axis, is rotated about the y-axis. The volume of the solid generated is given by

(A) $\pi \int_0^{\pi/2} y^2\, dy$ (B) $\pi \int_0^1 (\text{Arcsin}\, x)^2\, dx$ (C) $\pi \int_0^{\pi/2} (\text{Arcsin}\, x)^2\, dx$

(D) $\pi \int_0^{\pi/2} (\sin y)^2\, dy$ (E) $\pi \int_0^1 (\sin y)^2\, dy$

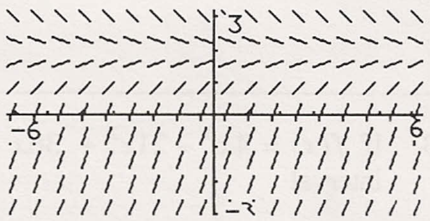
Ans

19. Find the coordinates of the absolute maximum point for the curve $y = xe^{-kx}$ where k is a fixed positive number.

(A) $\left(\frac{1}{k}, \frac{1}{ke}\right)$ (B) $\left(\frac{-1}{k}, \frac{-e}{k}\right)$ (C) $\left(\frac{1}{k}, \frac{1}{e^k}\right)$ (D) $(0, 0)$ (E) there is no maximum

Ans

20. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



(A) $\frac{dy}{dx} = 2 - \ln x$

(B) $\frac{dy}{dx} = 2 - e^{-x}$

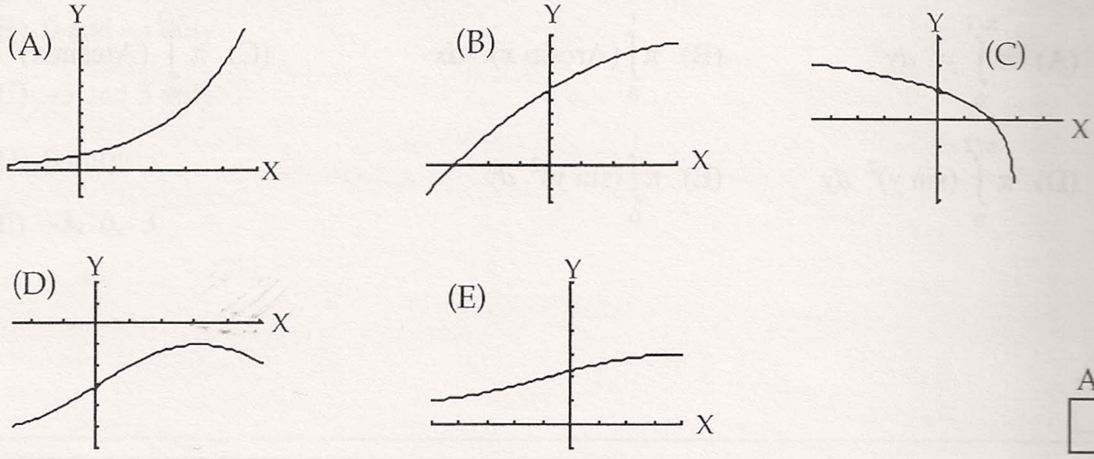
(C) $\frac{dy}{dx} = y - 2y^2$

(D) $\frac{dy}{dx} = 2 - y$

(E) $\frac{dy}{dx} = -x^2$

Ans

21. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



Ans

22. Use the Trapezoid Rule with $n = 3$ to approximate the area under $y = x^2$ from $x = 1$ to $x = 4$.

(A) $\frac{45}{3}$

(B) $\frac{43}{3}$

(C) $\frac{43}{2}$

(D) 43

(E) 21

Ans

23. If $f(x) = 4x^3 - 21x^2 + 36x - 4$, then the graph of f is decreasing and concave up on the interval

(A) $\left(\frac{3}{2}, 2\right)$ (B) $\left(-\infty, \frac{7}{4}\right)$ (C) $\left(\frac{7}{4}, \infty\right)$ (D) $\left(\frac{7}{4}, 2\right)$ (E) $\left(\frac{3}{2}, \frac{7}{4}\right)$

Ans

24. The number of bacteria in a culture is growing at a rate of $1500e^{3t/4}$ per unit of time t. At $t = 0$, the number of bacteria present was 2,000. Find the number present at $t = 4$.

(A) $2000\,e^3$

(B) $6000\,e^3$

(C) $2000\,e^6$

(D) $1500\,e^6$

(E) $1500\,e^3 + 500$

Ans

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$ and the line $x = 3m$, $m > 0$. The area of this region

(A) is independent of m

(B) increases as m increases

(C) decreases as m increases

(D) decreases for all $m < \frac{1}{3}$

(E) increases for all $m < \frac{1}{3}$

Ans

26. The formula $x(t) = \ln t + \frac{t^2}{18} + 1$ gives the position of an object moving along the x-axis during the time interval $1 \le t \le 5$. At the instant when the acceleration of the object is zero, the velocity is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) undefined

Ans

27. $\int 6 \sin x \cos^2 x \, dx =$

(A) $2\sin^3 x + C$

(B) $-2\sin^3 x + C$

(C) $2\cos^3 x + C$

(D) $-2\cos^3 x + C$

(E) $3\sin^2 x \cos^2 x + C$

Ans

28. If for all $x > 0$, $G(x) = \int_1^x \sin(\ln 2t)\, dt$, then the value of $G''\left(\frac{1}{2}\right)$ is

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) undefined

Ans

EXAM IV
CALCULUS AB
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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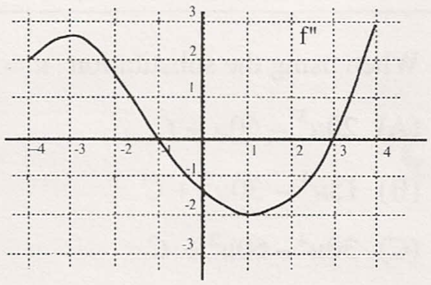
1. The graph of the **second derivative** of a function f is shown at the right. Which of the following is true?

the graph of f''



I. The graph of f has an inflection point at $x = -1$.

II. The graph of f is concave down on the interval $(-1, 3)$.

III. The graph of the derivative function f' is increasing at $x = 1$.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

Ans

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2. If the function f is continuous for all positive real numbers and if $f(x) = \dfrac{\ln x^2 - x\ln x}{x-2}$ when $x \neq 2$, then $f(2) =$

(A) -1 (B) -2 (C) $-e$ (D) $-\ln 2$ (E) undefined

Ans

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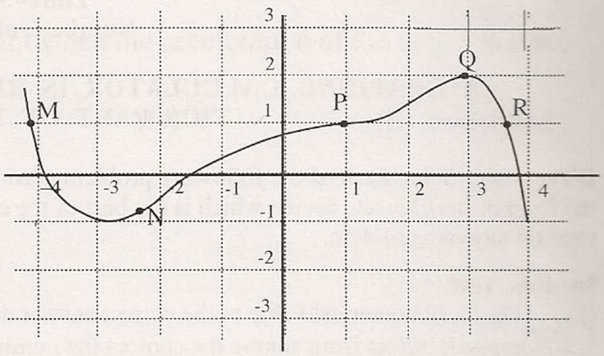
3. The graph of the function f is shown at the right. At which point on the graph of f are all the following true?

$f(x) > 0$, and $f'(x) < 0$ and $f''(x) < 0$

the graph of f



(A) M (B) N (C) P (D) Q (E) R

Ans

4. When using the substitution $u = \sqrt{1+x}$, an antiderivative of $f(x) = 60x\sqrt{1+x}$ is

(A) $20u^3 - 60u + C$

(B) $15u^4 - 30u^2 + C$

(C) $30u^4 - 60u^2 + C$

(D) $24u^5 - 40u^3 + C$

(E) $12u^6 - 20u^4 + C$

Ans

5. At $x = 0$, which of the following statements is TRUE of the function f defined by $f(x) = \sqrt{x^2 + .0001}$.

I. f is discontinuous II. f has a horizontal tangent III. f' is undefined

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, III

Ans

6. Functions f and g are defined by $f(x) = \frac{1}{x^2}$ and $g(x) = \arctan x$. What is the approximate value of x for which $f'(x) = g'(x)$?

(A) -3.36 (B) -2.86 (C) -2.36 (D) 1.36 (E) 2.36

Ans

7. The area of the region bounded below by $f(x) = x^2 - 7x + 10$ and above by $g(x) = \ln(x-1)$ is closest to

(A) 7.35
(B) 7.36
(C) 7.38
(D) 7.40
(E) 7.42

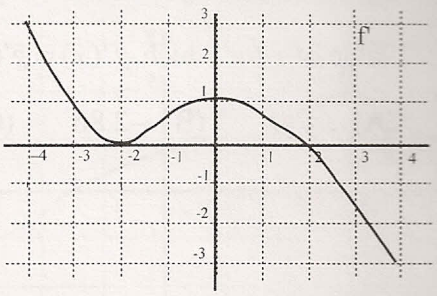
Ans

8. The average rate of change of the function $f(x) = \int_0^x \sqrt{1 + \cos(t^2)}\, dt$ over the interval $[1, 3]$ is nearest to

(A) 0.85
(B) 0.86
(C) 0.87
(D) 0.88
(E) 0.89

Ans

9. The graph of the *derivative* of f is shown at the right. Which of the following is true about the function f?



I. f is decreasing at $x = 0$.

II. f has a local maximum at $x = 2$.

III. The graph of f is concave up at $x = -1$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

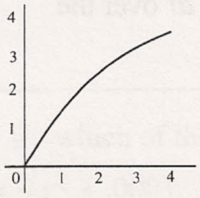
Ans

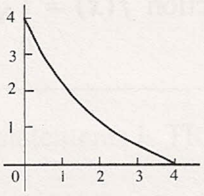
10. The total area enclosed between the graphs of $y = 3\cos x$ and $y = 1 - x$ is

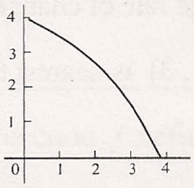
(A) 4.92
(B) 4.94
(C) 4.96
(D) 4.98
(E) 5.00

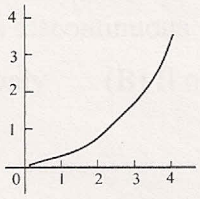
Ans

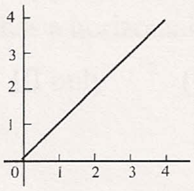
11. If a left Riemann sum overapproximates the definite integral $\int_0^4 f(x)\,dx$ and a trapezoid sum underapproximates the integral $\int_0^4 f(x)\,dx$, which of the following could be a graph of $y = f(x)$?

(A)


(B)


(C)


(D)


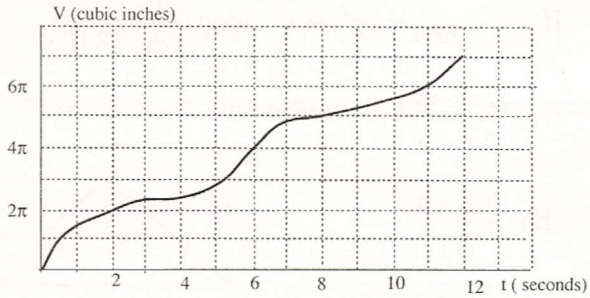
(E)


Ans

12. The function V whose graph is sketched below gives the volume of air, $V(t)$, (measured in cubic inches) that a man has blown into a balloon after t seconds.

$$\left(V = \frac{4}{3}\pi r^3\right)$$

The rate at which the radius is changing after 6 seconds is nearest to



(A) 0.05 in/sec (B) 0.12 in/sec (C) 0.21 in/sec (D) 0.29 in/sec (E) 0.37 in/sec

Ans

13. At how many points on the inteval $-2\pi \le x \le 2\pi$ does the tangent to the graph of the curve $y = x\cos x$ have slope $\frac{\pi}{2}$?

(A) 5

(B) 4

(C) 3

(D) 2

(E) 1

Ans

14. If $(x - y)^2 = y^2 - xy$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2x - y}{2y - x}$

(B) $\dfrac{2x - y}{2x}$

(C) $\dfrac{2x - y}{x}$

(D) $\dfrac{2x + 3y}{x}$

(E) undefined

Ans

15. Let the base of a solid be the first quadrant region bounded above by the graph of $y = \sqrt{x}$ and below by the x-axis on the interval [0, 4]. If every cross section perpendicular to the x-axis is an isosceles right triangle with one leg in the base, then the volume of the solid is

(A) 2

(B) 4

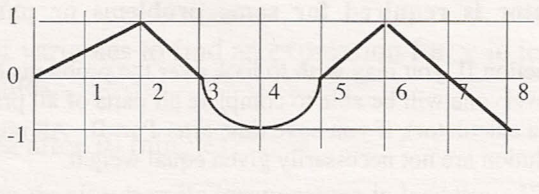
(C) 8

(D) 16

(E) 32

Ans

16. Let the function F be defined on the interval [0, 8] by $F(x) = \int_0^x f(t)\,dt$, where the graph of f is shown below. The graph of f consists of four line segments and a semicircle.



graph of $y = f(t)$

In which of the following intervals does F have a zero?

I. $4 < x < 5$ II. $5 < x < 6$ III. $6 < x < 7$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

Ans

17. The change in N, the number of bacteria in a culture dish at time t, is given by: $\frac{dN}{dt} = 2N$.

If $N = 3$ when $t = 0$, the approximate value of t when $N = 1210$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Ans

EXAM IV
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of problems–3

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let $y(t)$ be the temperature, in degrees Fahrenheit, of a cup of tea at time t minutes, $t \geq 0$. Room temperature is 70° and the initial temperature of the tea is 180°. The tea's temperature at time t is described by the differential equation $\frac{dy}{dt} = -0.1(y-70)$, with the initial condition $y(0) = 180$.

   (a) Use separation of variables to find an expression for y in terms of t, where t is measured in minutes.

   (b) How hot is the tea after 10 minutes?

   (c) If the tea is safe to drink when its temperature is less than 120°, at what time is the tea safe to drink?

2. Two points, A and B, are located 275 ft apart on a level field At a given instant, a balloon is released at B and rises vertically at a constant rate of 2.5 ft/sec, and, at the same instant, a cat starts running from A to B at a constant rate of 5 ft/sec.

(a) After 40 seconds, is the distance between the cat and the balloon decreasing or increasing? At what rate?

(b) Describe what is happening to the distance between the cat and the balloon at $t = 50$ seconds.

(c) Is there a time when the cat is closest to the balloon? If yes, find this time. If no, explain why?

3. Suppose the derivative of a polynomial function p is $p'(x) = (x+1)(x-1)(x-2)^2(x-4)^3$.

   (a) What is the degree of p?

   (b) What is the instantaneous rate of change of p at $x = 6$?

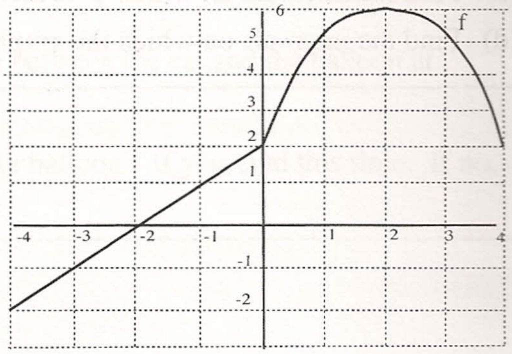
   (c) Find the intervals on which p is increasing.

   (d) Find the intervals on which the graph of p is concave down.

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THE TIMED PORTION FOR PART B, YOU MAY GO BACK AND CONTINUE TO WORK ON THE PROBLEMS IN PART A WITHOUT THE USE OF A CALCULATOR.

4. Answer the following questions about the function f, whose graph is shown at the right.

graph of the function f



(a) Find $\lim_{x \to 0} f(x)$.

(b) Find $\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$.

(c) Find $\lim_{x \to 0} f'(x)$.

(d) Find $\int_{-1}^{0} f(x)\, dx$.

(e) Approximate $\int_{-2}^{4} f(x)\, dx$ using the Trapezoid Rule with $n = 3$ subdivisions.

5. Let g be the function given by $g(x) = \dfrac{x \cdot |x|}{x^2 + 1}$.

   (a) Determine whether the *derivative* of the function g is even, odd, or neither.

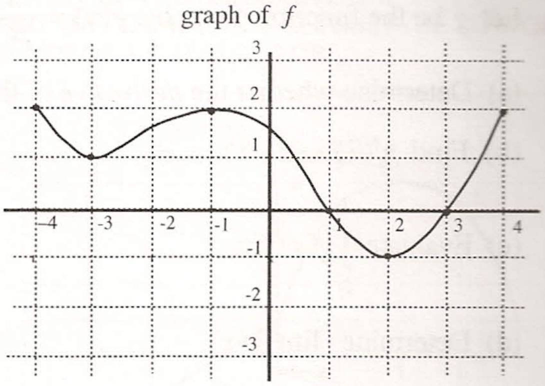
   (b) Find $g'(2)$.

   (c) Evaluate $\displaystyle\int_0^1 g(x)\,dx$.

   (d) Determine $\displaystyle\lim_{x \to \infty} g(x)$.

   (e) Find the range of g. Justify your answer.

6. The graph of a differentiable function f on the closed interval $[-4, 4]$ is shown at the right.



Let $G(x) = \int_{-4}^{x} f(t)\, dt$ for $-4 \leq x \leq 4$.

(a) Find $G(-4)$.

(b) Find $G'(-1)$.

(c) On which interval or intervals is the graph of G concave down. Justify your answer.

(d) Find the value of x at which G has its maximum on the closed interval $[-4, 4]$. Justify your answer.